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# $N = 1/2$ Supergravity with Matter in Four Euclidean Dimensions <sup>1</sup>

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## Abstract

An  $N = 1/2$  supergravity in four Euclidean spacetime dimensions, coupled to both vector- and scalar-multiplet matter, is constructed for the first time. We begin with the standard  $N = (1, 1)$  conformally extended supergravity in four Euclidean dimensions, and freeze out the graviphoton field strength to an arbitrary (fixed) self-dual field (the so-called  $C$ -deformation). Though a consistency of such procedure with local supersymmetry is not guaranteed, we find a simple consistent set of algebraic constraints that reduce the local supersymmetry by  $3/4$  and eliminate the corresponding gravitini. The final field theory (after the superconformal gauge-fixing) has the residual local  $N = (0, \frac{1}{2})$  or just  $N = 1/2$  supersymmetry with only one chiral gravitino as the corresponding gauge field. Our theory is not ‘Lorentz’-invariant because of the non-vanishing self-dual graviphoton vacuum expectation value, which is common to the  $C$ -deformed  $N = 1/2$  rigidly supersymmetric field theories constructed in a non-anticommutative superspace.

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# 1 Introduction

A construction of new supergravity theories is apparently complete after a lot of work done in the past — see e.g., refs. [1, 2]. However, it is merely apparent, because some recent developments in field theory, strings and gravity offer new opportunities for even further generalizations of supergravities, by relaxing some of the symmetry requirements. It offers new perspectives to various physical applications, such as (i) partial supersymmetry breaking, and (ii) brane supersymmetry reduction.

One of the recent developments is a noncommutative gravity. Though the idea of replacing the ordinary field product by the noncommutative Moyal (star) product is not new [3], its implementation is not unique, while it often leads to a complexification of the metric and the appearance of ghosts (see, however, refs. [4] for possible cures). The appearance of infinitely many interaction vertices with unlimited powers of momenta is the necessary feature of those noncommutative gravity models.

String theory can teach us more about noncommutative gravity (see e.g., ref. [5]), as well as about supersymmetry [6]. In particular, as was observed by Ooguri and Vafa [6], the superworldvolume of a supersymmetric D-brane in a constant Ramond-Ramond type flux gives rise to the remarkable new structure in the corresponding superspace, which is now called Non-AntiCommutativity (NAC). It means that the fermionic superspace coordinates are no longer Grassmann (i.e. they no longer anti-commute), but satisfy a Clifford algebra. In other words, the impact of the Ramond-Ramond flux on the D-brane dynamics can be described by the non-anticommutativity in the D-brane superworldvolume.

As regards a D3-brane, a 10-dimensional (self-dual) five-form flux upon compactification to four dimensions gives rise to the (self-dual) graviphoton flux in the D3-brane 4-dimensional worldvolume [6]. In its turn, the non-anticommutativity in superspace can be described by the (Moyal-Weyl type) non-anticommutative star product amongst superfields. It results in a construction of the NAC deformed supersymmetric field theories with partially broken supersymmetry, pioneered by Seiberg [7], in four Euclidean dimensions. Unlike bosonic noncommutativity, the NAC supersymmetric field theories usually have only a limited number of new interaction terms, without higher derivatives, while their Lagrangians can often be written down in closed form. As a matter of fact, all recent studies of the NAC supersymmetric field theories, following ref. [7], were limited to *rigid* supersymmetry, i.e. without gravity or supergravity (see e.g., ref. [8] and references therein).

Given the relation between a non-anticommutativity and a non-vanishing (self-dual) vacuum expectation value of a graviphoton in four dimensions, it is quite natural to explore further possibilities for a construction of new supergravities, by freezing a graviphoton field in an extended supergravity theory (with

matter). The minimally extended Poincaré supergravity in four dimensions, that has a graviphoton as the superpartner of a graviton, is the  $N = (1, 1)$  or just  $N = 2$  supergravity. The structure of  $N = 2$  supergravity with matter was given in detail in refs. [9, 10].

In our earlier paper [11], a toy model of the four-dimensional  $N = 1/2$  supergravity with a fixed self-dual graviphoton expectation value was constructed by freezing out the graviphoton field strength in the standard  $N = (1, 1)$  extended supergravity with two non-chiral gravitini [12]. Our supergravity model [11] has local  $N = (0, \frac{1}{2})$  supersymmetry. Consistency of the model [11] requires the expectation value of the graviphoton field strength to be equal to the self-dual (bilinear) gravitino condensate.

An extension of the construction [11] to a matter-coupled  $N = 1/2$  supergravity is not automatic since more consistency conditions have to be satisfied. In this paper we report our results of such construction, by presenting the Lagrangian and the local supersymmetry transformation laws of the  $N = 1/2$  supergravity in four Euclidean dimensions, coupled to vector supermultiplets and scalar supermultiplets, with all the fermionic terms included.

Our paper is organized as follows: in Sec. 2 we briefly describe the contents of  $N = 2$  conformal supergravity multiplets in the  $N = 2$  superconformal tensor calculus [9]. In Sec. 3 we introduce the 2-component notation for spinors in four Euclidean dimensions. In Sec. 4 we describe our way of freezing of a graviphoton field, by imposing a self-duality condition on the graviphoton field strength, and then studying its consistency and the surviving symmetries.<sup>4</sup> The residual superconformal transformations are given in Sec. 5. The superconformal gauge-fixing and elimination of the auxiliary fields are discussed in Sec. 6. Our Lagrangian of the  $N = 1/2$  Euclidean supergravity with matter is given in Sec. 7. Sec. 8 is our Conclusion. Three Appendices are devoted to further notation and some technical details. The notation [9] for hypermultiplets is briefly summarized in Appendix A. In Appendix B we collect the  $N = 2$  superconformal transformation laws [9] which is the starting point of our construction. In Appendix C we quote the so-called decomposition rules [9] needed in passing from the conformal supergravity to the ‘Poincaré’ supergravity.

## 2 $N = 2$ supergravity field components

The last paper of ref. [9] is the pre-requisite to our construction. So, instead of copying the equations of ref. [9] here, we merely review the methods of ref. [9], and concentrate on the differences between our construction and that of ref. [9].

First, our construction can only be defined in *Euclidean* four-dimensions, not in Minkowski spacetime as in ref. [9]. As is well known, the change of signature

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<sup>4</sup>As regards a consistent reduction of the  $N = 2$  matter-coupled supergravity to  $N = 1$  matter-coupled supergravity, including all fermionic terms, see ref. [13].

has important implications on the structure of field representations, especially on spinors. For instance, minimal spinor representations in Minkowski signature are given by real (Majorana) spinors or complex chiral spinors, while the chiral and anti-chiral parts of a Majorana spinor are related by complex conjugation. Accordingly, a number of supersymmetries in Minkowski spacetime is measured by a number of Majorana supercharges. In four Euclidean dimensions, Majorana spinors do not exist [15], whereas the chiral and anti-chiral spinors are independent. Hence, the numbers of left and right (chiral and anti-chiral) Euclidean supercharges need not be the same, while the minimal choice is obviously given by one chiral or anti-chiral supercharge. We call it  $(1/2, 0)$  or  $(0, 1/2)$  susy, respectively, or simply  $N = 1/2$  supersymmetry.

Second, in order to make the chiral supersymmetry manifest, we are going to use the 2-component notation for spinors, which is best suitable for our purposes. So we rewrite the results of ref. [9] obtained in the 4-component spinor notation, to the 2-component notation in four Euclidean dimensions (see Sec. 3).

The  $N = 2$  superconformal tensor calculus gives us a systematic method for constructing the  $N = 2$  super-conformal and super-Poincaré-invariant Lagrangians and the  $N = 2$  transformation laws (see e.g., ref. [14] for a review.) It provides us with (i) the off-shell  $N = 2$  supermultiplets, as the representations of  $N = 2$  local superconformal algebra, together with the transformation laws of their field components, which form a closed algebra, (ii) the multiplication rules for a construction of new representations, and (iii) the density formulas describing the superconformal invariants.

In order to get the super-Poincaré Lagrangian and the transformation laws, one has to fix the truly superconformal symmetries, while keeping the super-Poincaré ones. It is often called ‘gauge fixing’. The gauge fixing conditions give rise to the decomposition laws relating the truly superconformal transformation parameters to the super-Poincaré transformation parameters — see refs. [9, 14] for details.

The off-shell  $N = 2$  superconformal multiplets are given by [9]

- a *Weyl* multiplet,
- a *vector* multiplet,
- a *hypermultiplet*.

The  $N = 2$  *Weyl* multiplet has  $24 + 24$  independent field components:

$$(e_\mu^a, \psi_\mu^i, b_\mu, A_\mu, \mathcal{V}_\mu^i{}_j, T_{ab}{}^{ij}, \chi^i, D) \quad (2.1)$$

where  $e_\mu^a$  is vierbein, and the gravitino doublet  $\psi_\mu^i$  is the gauge field of  $N = 2$  local supersymmetry. The gauge fields of other superconformal symmetries are  $b_\mu$  for dilatations,  $A_\mu$  for chiral  $U(1)$  rotations, and  $\mathcal{V}_\mu^i{}_j$  for chiral  $SU(2)$  rotations.

We also need the auxiliary fields: the bosonic tensor  $T_{ab}^{ij}$  and the real scalar  $D$ , and a fermionic (spinor) doublet  $\chi^i$ .

Only  $e_\mu^a$  and  $\psi_\mu^i$  are going to represent physical degrees of freedom, the  $\mathcal{V}_\mu^i{}_j$  is the antihermitian traceless matrix in its  $SU(2)$  indices  $i, j$ , while  $T_{ab}^{ij}$  is the real tensor antisymmetric in its both  $SU(2)$  and ‘Lorentz’ index pairs.

An  $N = 2$  *vector* multiplet has  $8 + 8$  independent field components:

$$(X, \Omega_i, W_\mu, Y_{ij}) \quad (2.2)$$

where  $X$  is a complex scalar,  $\Omega_i$  is a spinor doublet,  $W_\mu$  is a vector gauge field, and  $Y_{ij}$  is a real  $SU(2)$  auxiliary triplet.

We consider the vector gauge fields to be Lie-algebra valued, with the hermitean generators  $t_I$  obeying an algebra

$$[t_I, t_J] = f_{IJ}{}^K t_K \quad (2.3)$$

where we have introduced the Lie algebra structure constants  $f_{IJ}{}^K$ . So, the vector multiplet components carry the extra (gauge) index,  $I, J, \dots = 0, 1, \dots, n$ . We choose  $I = 0$  for a graviphoton (abelian) gauge field, so we also define  $\tilde{I}, \tilde{J}, \dots = 1, 2, \dots, n$ .

The *hypermultiplet* physical fields are given by

$$(A_i^\alpha, \zeta^\alpha) \quad (2.4)$$

where  $A_i$  is a scalar doublet and  $\zeta$  is a complex spinor. The hypermultiplets are supposed to belong to a representation of the non-abelian gauge group. The index  $i = 1, 2$  is associated to the  $SU(2)$  automorphism group of the  $N = 2$  supersymmetry algebra, whereas the index  $\alpha = 1, 2, \dots, 2r$  is the representation index with respect to the non-abelian gauge group. See Appendix A for more about the hypermultiplet notation.

The  $N = 2$  superconformal transformation laws in the 2-component Euclidean notation are collected in Appendix B.

A consistent Wick rotation of a field theory with fermions from four Minkowski dimensions to four Euclidean dimensions is described in detail in ref. [15]. In ref. [9] the spacetime signature  $(+++ -)$  was used, which is now going to be Wick-rotated to  $(++++)$  by setting  $x_4 = it$ . As regards the vector gauge fields, it implies  $A_\mu \rightarrow (\vec{A}^E, iA_4^E)$ . As is argued in ref. [15], one should change  $\gamma_4 \rightarrow i\gamma_E^5$ , and use gamma matrices and a charge conjugation matrix in four Euclidean dimensions (see e.g., an Appendix in ref. [1]).<sup>5</sup> So our definition of the Dirac conjugation is

$$\bar{\lambda}^i = (\lambda_i)^\dagger i\gamma_E^5. \quad (2.5)$$

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<sup>5</sup>In ref. [15] the Dirac conjugation includes a factor of  $i$ , while the Lagrangian excludes a factor of  $i$ , but we are going to use the opposite notation.

For instance, the Majorana condition is modified as follows:

$$(\bar{\lambda}^i)^T C_E = (\lambda_i)^\dagger i \gamma_E^5 \quad (2.6)$$

It is worth mentioning that this condition is *not* a reality condition for spinors. Nevertheless, we can still use this condition for constructing the Euclidean version of a given supergravity theory. To avoid confusion, we sometimes append a script (E) for Euclidean fields or matrices, and a script (M) for their Minkowski counterparts.

### 3 Euclidean 2-component spinor notation

We use lower case Greek letters for curved space vector indices,  $\mu, \nu, \dots = 1, 2, 3, 4$ , lower case Latin indices for flat (tangent) space vector indices,  $a, b, \dots = 1, 2, 3, 4$ , and capital Latin letters for (anti)chiral spinor indices (dotted or undotted),  $A, B, \dots = 1, 2$ .

Gamma matrices in four Euclidean dimensions satisfy an algebra

$$\{\gamma_a, \gamma_b\} = 2\delta_{ab}, \quad \{\gamma_5, \gamma_a\} = 0 \quad (3.1)$$

An explicit representation of the Euclidean gamma matrices is as follows [1]:

$$\gamma_k = \begin{pmatrix} 0 & -i\sigma_k^{A\dot{B}} \\ i\sigma_{k\dot{A}B} & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_{5E} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (3.2)$$

where  $\sigma_k$  are Pauli matrices,  $k = 1, 2, 3$ . In addition, we define the matrices

$$\sigma^{ab} = \frac{1}{4} \begin{pmatrix} (\sigma^a \sigma^b - \sigma^b \sigma^a)^{A\dot{C}} & 0 \\ 0 & (\sigma^a \sigma^b - \sigma^b \sigma^a)_{\dot{A}C} \end{pmatrix} =: \begin{pmatrix} \sigma^{abA}{}_{\dot{C}} & 0 \\ 0 & \sigma^{ab}{}_{\dot{A}C} \end{pmatrix} \quad (3.3)$$

which are anti-hermitean,

$$(\sigma_{ab})^\dagger = -\sigma_{ba} \quad , \quad (3.4)$$

in terms of their self-dual and anti-self-dual combinations,

$$\frac{1}{2}\varepsilon^{abcd}\sigma_{cd\dot{A}}{}^{\dot{B}} = \sigma^{ab}{}_{\dot{A}}{}^{\dot{B}} \quad \text{and} \quad \frac{1}{2}\varepsilon^{abcd}\sigma_{cd}{}^A{}_B = -\sigma^{abA}{}_B \quad (3.5)$$

The Euclidean charge conjugation matrix is given by

$$C = \gamma_4 \gamma_2 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix} = \begin{pmatrix} \epsilon_{AB} & 0 \\ 0 & \epsilon^{\dot{A}\dot{B}} \end{pmatrix} \quad (3.6)$$

so that

$$C = -C^T, \quad C\gamma^a C^{-1} = -(\gamma^a)^T, \quad (3.7)$$

$$C^\dagger = C^{T*} = -C^*, \quad C^* = \begin{pmatrix} \epsilon^{AB} & 0 \\ 0 & \epsilon_{\dot{A}\dot{B}} \end{pmatrix}, \quad (3.8)$$

$$C^*C = CC^* = -I \quad (3.9)$$

When changing the notation [9] by representing all the 4-component spinors in terms of their 2-component constituents in Euclidean space, it is important to observe the chirality of each 4-component spinor given by the position (up or down) of its  $SU(2)$  index ( $i, j = 1, 2$ ), e.g.,

$$\Omega_i^I =: \begin{pmatrix} \Omega_i^{IB} \\ 0 \end{pmatrix}, \quad \Omega^{iI} =: \begin{pmatrix} 0 \\ \bar{\Omega}^{iI}_{\dot{B}} \end{pmatrix}, \quad (3.10)$$

$$\bar{\Omega}_M^{iI} \rightarrow (\Omega_E^{iI})^T C_E =: (0, -\bar{\Omega}^{iI\dot{A}}) , \quad \bar{\Omega}_{Mi}^I \rightarrow (\Omega_{Ei}^I)^T C_E =: (\Omega_{Ai}^I, 0) . \quad (3.11)$$

and

$$\psi_\mu^i =: \begin{pmatrix} \psi_\mu^{iB} \\ 0 \end{pmatrix}, \quad \psi_{\mu i} =: \begin{pmatrix} 0 \\ \bar{\psi}_{\mu i \dot{B}} \end{pmatrix}, \quad (3.12)$$

$$\bar{\psi}_{\mu M}^i \rightarrow (\psi_{\mu E}^i)^T C_E =: (\psi_{\mu A}^i, 0), \quad \bar{\psi}_{\mu i M} \rightarrow (\psi_{\mu i E})^T C_E =: (0, -\bar{\psi}_{\mu i}^{\dot{A}}) \quad (3.13)$$

In order to avoid double counting, their complex conjugates are given by

$$(\Omega^{iI})^* = \begin{pmatrix} -i\Omega_{iA}^I \\ 0 \end{pmatrix}, \quad (\Omega^{iI})^\dagger = (-i\Omega_{iA}^I, 0), \quad (3.14)$$

$$(\Omega_i^I)^* = \begin{pmatrix} 0 \\ -i\bar{\Omega}^{iI\dot{A}} \end{pmatrix}, \quad (\Omega_i^I)^\dagger = (0, -i\bar{\Omega}^{iI\dot{A}}), \quad (3.15)$$

$$(\psi_\mu^i)^* = \begin{pmatrix} 0 \\ -i\bar{\psi}_{\mu i}^{\dot{A}} \end{pmatrix}, \quad (\psi_\mu^i)^\dagger = (0, -i\bar{\psi}_{\mu i}^{\dot{A}}), \quad (3.16)$$

$$(\psi_{\mu i})^* = \begin{pmatrix} -i\psi_{\mu A}^i \\ 0 \end{pmatrix}, \quad (\psi_{\mu i})^\dagger = (-i\psi_{\mu A}^i, 0) \quad (3.17)$$

The  $SU(2)$  indices are contracted by  $\varepsilon_{ij}$  and  $\varepsilon^{ij}$ ,

$$\varepsilon_{ij}\varepsilon^{ij} = 2, \quad \varepsilon_{12} = -\varepsilon_{21} = \varepsilon^{12} = -\varepsilon^{21} = 1, \quad (\varepsilon_{ij})^\dagger = \varepsilon^{ik}\varepsilon^{jl}\varepsilon_{kl} = \varepsilon^{ij} \quad (3.18)$$

Two-component spinor indices are contracted as follows:

$$\psi_A = \psi^B \epsilon_{BA}, \quad \psi^A = \epsilon^{AB} \psi_B, \quad \bar{\psi}_{\dot{A}} = \bar{\psi}^{\dot{B}} \epsilon_{\dot{B}\dot{A}}, \quad \bar{\psi}^{\dot{A}} = \epsilon^{\dot{A}\dot{B}} \bar{\psi}_{\dot{B}}, \quad (3.19)$$

$$\sigma_{a\dot{D}C} = \sigma_a^{A\dot{B}} \epsilon_{AC} \epsilon_{\dot{B}\dot{D}}, \quad \sigma_a^{A\dot{B}} = \epsilon^{\dot{B}\dot{D}} \epsilon^{AC} \sigma_{a\dot{D}C}, \quad (3.20)$$

where we have

$$\epsilon_{AB} = -\epsilon_{BA}, \quad \epsilon^{AB} \epsilon_{BC} = -\delta^A_C, \quad \epsilon_{12} = \epsilon^{12} = -\epsilon_{\dot{1}\dot{2}} = -\epsilon^{\dot{1}\dot{2}} = 1 \quad (3.21)$$

We use the following book-keeping notation:

$$\begin{aligned} \lambda \sigma_a \bar{\chi} &\equiv \lambda_A \sigma_a^{A\dot{B}} \bar{\chi}_{\dot{B}}, & \lambda \sigma^a \sigma^b \chi &\equiv \lambda_A \sigma^{aAB} \sigma_{\dot{B}C}^b \chi^C, & \lambda \chi &\equiv \lambda^D \chi_D, \\ \bar{\lambda} \sigma_a \chi &\equiv \bar{\lambda}^{\dot{A}} \sigma_{a\dot{A}B} \chi^B, & \bar{\lambda} \sigma^a \sigma^b \bar{\chi} &\equiv \bar{\lambda}^{\dot{A}} \sigma_{\dot{A}B}^a \sigma^{b\dot{C}C} \bar{\chi}_{\dot{C}}, & \bar{\lambda} \bar{\chi} &\equiv \bar{\lambda}^{\dot{D}} \bar{\chi}_{\dot{D}}. \end{aligned} \quad (3.22)$$

A fully antisymmetric Levi-Civita symbol  $\varepsilon^{abcd}$  is normalized by  $\varepsilon^{1234} = 1$ . The (anti-)self-dual parts of an antisymmetric tensor  $T_{ab}$  are <sup>6</sup>

$$\tilde{T}_{ab} = \frac{1}{2} \varepsilon_{abcd} T^{cd}, \quad T_{ab}^{\pm} = \frac{1}{2} (T_{ab} \pm \tilde{T}_{ab}) \quad (3.23)$$

## 4 Consistent freezing of a graviphoton field

Our basic idea is to eliminate a graviphoton field from the  $N = 2$  matter-coupled supergravity, by assigning it a fixed value, say, its vacuum expectation value (VEV). Generally speaking, it is going to break supersymmetry, because the graviphoton is a field component of  $N = 2$  supergravity multiplet. We show in this section that, when assigning a self-dual vacuum expectation value to a graviphoton, the  $N = 2$  local supersymmetry can be consistently broken to an  $N = 1/2$  local supersymmetry in four Euclidean dimensions, and in the presence of a generic  $N = 2$  matter, thus generalizing the earlier results [11] obtained for pure  $N = 2$  supergravity without matter.

The graviphoton field is identified with  $W_{\mu}^0(x)$  after imposing the gauge condition on  $X^0$  by using the local chiral  $U(1)$  rotations [9]:

$$X^0 = \bar{X}^0 > 0 \quad (4.1)$$

We prefer to impose the gauge condition (4.1) *after* our truncation procedure described in this section. The graviphoton field strength is given by

$$F_{\mu\nu}^0 = \partial_{\mu} W_{\nu}^0 - \partial_{\nu} W_{\mu}^0 \quad (4.2)$$

We now freeze the graviphoton field by imposing the self-dual constraints:

$$F_{\mu\nu}^{0+} \equiv C_{\mu\nu}(x) \quad \text{and} \quad F_{\mu\nu}^{0-} \equiv 0, \quad (4.3)$$

where  $C_{\mu\nu}(x)$  is a fixed self-dual antisymmetric tensor field (VEV) with an arbitrary  $x$ -dependence.

Our strategy is to look for the *residual* local supersymmetry that keeps our assignment (4.3) invariant, i.e. that leaves both  $F_{\mu\nu}^{0+}$  and  $F_{\mu\nu}^{0-}$  to be unchanged. As was shown in ref. [11] in the case of the  $N = 2$  supergravity without matter, we already have to fix 3/4 of  $Q$ -supersymmetry by eliminating three (out of four) infinitesimal local  $Q$ -supersymmetry chiral spinor parameters as follows:

$$\varepsilon^1 \equiv \varepsilon^2 \equiv \bar{\varepsilon}_2 \equiv 0 \quad (4.4)$$

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<sup>6</sup>Our sign convention here is opposite to that in refs. [1] and [11].



Then we may only hope that the remaining local  $N = 1/2$  supersymmetry with the chiral spinor parameter  $\bar{\varepsilon}_1(x)$  remains to be a symmetry of the theory after investigating all the consistency conditions originating from the  $C$ -deformation (4.3):

$$\delta_{\bar{\varepsilon}_1} F_{\mu\nu}^0 = 0 \quad (4.5)$$

The  $N = 1/2$   $Q$ -supersymmetry transformation law of  $W_\mu^0$  can be read off from eq. (B.6):

$$\delta_{\bar{\varepsilon}_1} W_\mu^0 = -i\bar{\varepsilon}_1 \sigma_\mu \Omega_2^0 - 2\bar{\varepsilon}_1 \bar{\psi}_{\mu 2} X^0 \quad (4.6)$$

It implies the following consistency condition:

$$(\sigma_\mu \Omega_2^0)_{\dot{A}} - 2i\bar{\psi}_{\mu 2 \dot{A}} X^0 = 0 \quad (4.7)$$

This algebraic condition can be easily solved, thus eliminating an independent gravitino field  $\bar{\psi}_{\mu 2 \dot{A}}$ , in terms of the other (matter) fields.

Equation (4.7) is also not invariant under the  $N = 1/2$   $Q$ -supersymmetry, so that we have to impose its invariance for consistency. By using the  $N = 1/2$   $Q$ -supersymmetry transformation laws

$$\delta_{\bar{\varepsilon}_1} e_\mu^m = -i\bar{\varepsilon}_1 \sigma^m \psi_\mu^1, \quad \delta_{\bar{\varepsilon}_1} \Omega_2^{0A} = 0, \quad \delta_{\bar{\varepsilon}_1} \bar{\psi}_{\mu 2 \dot{A}} = \mathcal{V}_{\mu 2}^1 \bar{\varepsilon}_{1 \dot{A}}, \quad \delta_{\bar{\varepsilon}_1} X^0 = 0, \quad (4.8)$$

we find from eqs. (B.1), (B.3), (B.4) and (B.7) that the constraint (4.7) yields yet another constraint

$$\psi_\mu^1 \Omega_2^0 - \mathcal{V}_{\mu 2}^1 X^0 = 0 \quad (4.9)$$

This is again an algebraic equation, while it can be easily solved for  $\mathcal{V}_{\mu 2}^1$ , thus eliminating that field in terms of the other fields.

Generally speaking, further consistency requirements might lead to the infinite and increasingly complicated set of constraints, but it does not happen in our case! We find that eq. (4.9) is invariant under the  $N = 1/2$   $Q$ -supersymmetry because of

$$\delta_{\bar{\varepsilon}_1} \psi_\mu^1 = 0 \quad \text{and} \quad \delta_{\bar{\varepsilon}_1} \mathcal{V}_{\mu 2}^1 = 0 \quad (4.10)$$

which easily follow from eqs. (B.2) and (B.16), respectively.

Having found the consistent short (finite) set of algebraic constraints that are invariant under the  $N = 1/2$  local  $Q$ -supersymmetry, we can easily check what are the other residual symmetries in the list of the  $N = 2$  superconformal transformation laws (see Appendices B and C), which also leave the constraints invariant. We then find that all our constraints (4.3), (4.7) and (4.9) are still invariant under the full  $S$ -supersymmetry and the chiral  $U(1)$  transformations, whereas the local  $SU(2)$  automorphisms of the  $N = 2$  supersymmetry algebra are broken:  $\Lambda^i_j = \Lambda_i^j = 0$ .

The residual  $S$ -supersymmetry implies that the original decomposition laws (see ref. [9] and Appendix C) are still valid, being subject to the conditions  $\varepsilon^1 = \varepsilon^2 = \bar{\varepsilon}_2 = 0$ . It means that our construction does not affect the  $S$ -gauge,

$K$ -gauge and  $D$ -gauge, as described in ref. [9]. So we are going to use the same gauges when passing to the Poincaré supergravity.

The rest of our construction of the  $N = 1/2$  matter-coupled supergravity is pretty straightforward (though tedious!), by inserting the equations derived in this section into the results of ref. [9], making gauge-fixing, and deriving the transformation rules and the Lagrangian of the (Poincaré)  $N = 1/2$  supergravity with matter. Our results are summarized in the next Secs. 5, 6 and 7.

## 5 $N = 1/2$ supergravity transformation laws

Here we summarize the residual local superconformal symmetry transformation laws of the independent field components with respect to the  $Q$ -supersymmetry with the parameters  $(\bar{\varepsilon}_1)$ , the  $S$ -supersymmetry with the parameters  $(\eta_i)$  and  $(\bar{\eta}^i)$ , and the chiral  $U(1)$  symmetry with the parameter  $(\Lambda_A)$ .

(i) As regards a vierbein and gravitini, we find

$$\begin{aligned}\delta e_\mu^a &= -i\bar{\varepsilon}_1\sigma^a\psi_\mu^1, \\ \delta\psi_\mu^{1A} &= +i(\sigma_\mu\bar{\eta}^1)^A - \frac{1}{2}i\Lambda_A\psi_\mu^{1A}, \\ \delta\psi_\mu^{2A} &= -\frac{1}{2}iT_{\mu m}^{-21}(\sigma^m\bar{\varepsilon}_1)^A + i(\sigma_\mu\bar{\eta}^2)^A - \frac{1}{2}i\Lambda_A\psi_\mu^{2A}, \\ \delta\bar{\psi}_{\mu 1\dot{A}} &= +2[\partial_\mu\bar{\varepsilon}_{1\dot{A}} - \frac{1}{2}\omega_\mu^{mn}\sigma_{mn\dot{A}}^{\dot{B}}\bar{\varepsilon}_{1\dot{B}} - \frac{1}{2}i\bar{A}_\mu\bar{\varepsilon}_{1\dot{A}}] + \mathcal{V}_{\mu 1}^1\bar{\varepsilon}_{1\dot{A}} \\ &\quad - i(\sigma_\mu\eta_1)_{\dot{A}} + \frac{1}{2}i\Lambda_A\bar{\psi}_{\mu 1\dot{A}}\end{aligned}$$

As is clear from those equations, the gravitino field  $\bar{\psi}_{\mu 1\dot{A}}$  is the gauge field of the residual  $N = 1/2$  local supersymmetry.

(ii) As regards the vector multiplet components, we find ( $I > 0$ )

$$\begin{aligned}\delta X^I &= -i\Lambda_A X^I, \\ \delta\bar{X}^I &= +\bar{\varepsilon}_1\bar{\Omega}^{1I} + i\Lambda_A\bar{X}^I, \\ \delta W_\mu^I &= -i\bar{\varepsilon}_1\sigma_\mu\Omega_2^I - 2\bar{\varepsilon}_1\bar{\psi}_{\mu 2}X^I, \\ \delta\Omega_1^{IA} &= -2i(\sigma^\lambda\bar{\varepsilon}_1)^A[(\partial_\lambda + iA_\lambda)X^I - gf_{JK}^IW_\lambda^JX^K - \frac{1}{2}\psi_\lambda^k\Omega_k^I] \\ &\quad + 2X^I\eta_1^A - \frac{1}{2}i\Lambda_A\Omega_1^{IA}, \\ \delta\Omega_2^{IA} &= +2X^I\eta_2^A - \frac{1}{2}i\Lambda_A\Omega_2^{IA}, \\ \delta\bar{\Omega}^{1I\dot{A}} &= +Y^{11I}\bar{\varepsilon}_1^{\dot{A}} + 2\bar{X}^I\bar{\eta}^{1\dot{A}} + \frac{1}{2}i\Lambda_A\bar{\Omega}^{1I\dot{A}}, \\ \delta\bar{\Omega}^{2I\dot{A}} &= +Y^{21I}\bar{\varepsilon}_1^{\dot{A}} + \bar{\varepsilon}_1^{\dot{B}}\sigma^{mn}{}_{\dot{B}}^{\dot{A}}\mathcal{F}_{mn}^{+I} + 2gf_{JK}^I\bar{X}^JX^K\bar{\varepsilon}_1^{\dot{A}} \\ &\quad + 2\bar{X}^I\bar{\eta}^{2\dot{A}} + \frac{1}{2}i\Lambda_A\bar{\Omega}^{2I\dot{A}}\end{aligned}$$

(iii) As regards the hypermultiplet components, we find (see Appendix A for the

notation)

$$\begin{aligned}
\delta A_1^\alpha &= -2\bar{\zeta}^\alpha \bar{\varepsilon}_1 , \\
\delta A_2^\alpha &= 0 , \\
\delta A^1_\alpha &= 0 , \\
\delta A^2_\alpha &= +2\rho_{\alpha\beta} \bar{\varepsilon}_1 \bar{\zeta}^\beta , \\
\delta \zeta_{\alpha A} &= +i(\bar{\varepsilon}_1 \sigma^\mu)_A \bar{D}_\mu A^1_\alpha + A^i_\alpha \eta_{iA} + \frac{i}{2} \Lambda_A \zeta_{\alpha A} , \\
\delta \bar{\zeta}^\alpha_{\dot{A}} &= -2g X^\alpha_{\beta} A_2^{\beta} \bar{\varepsilon}_{1\dot{A}} + A_i^\alpha \bar{\eta}^i_{\dot{A}} - \frac{i}{2} \Lambda_A \bar{\zeta}^\alpha_{\dot{A}}
\end{aligned}$$

where we have used the abbreviations (B.13), (B.14) and (B.15).

## 6 Gauge-fixing and eliminating auxiliary fields

In the superconformal tensor calculus, the  $N = 1$  or  $N = 2$  matter-coupled (Poincaré) supergravity is obtained from the  $N = 1$  or  $N = 2$  conformal supergravity, respectively, by imposing certain gauges, in order to fix the truly superconformal symmetries, while keeping the super-Poincaré symmetries. In this process the residual supersymmetry transformations are deformed by the compensating transformations needed to restore the gauges [9]. We follow here the same pattern by eliminating the truly superconformal  $N = 1/2$  symmetries after imposing our set of constraints (Sec. 4), with the help of the gauges similar to that of ref. [9]. Then we eliminate the auxiliary fields by using their algebraic equations of motion. The Lagrangian of the resulting  $N = 1/2$  supergravity is given in the next section.

- The  $K$ -gauge to fix the conformal boosts is given by

$$b_\mu = 0 \quad (6.1)$$

- The  $D$ -gauge to fix dilatations (and to get the standard normalization of the Einstein term in the Lagrangian) is given by

$$N_{IJ} X^I \bar{X}^J = 1, \quad A_i^\alpha d_\alpha^\beta A^i_\beta = -2 \quad (6.2)$$

- The  $S$ -gauge to fix  $S$ -supersymmetry is

$$X^I N_{IJ} \bar{\Omega}^{iJ}_{\dot{A}} = 0, \quad \bar{X}^I N_{IJ} \Omega_i^{JA} = 0, \quad A_i^\alpha d_\alpha^\beta \bar{\zeta}^\beta_{\dot{A}} = 0, \quad \zeta^A_\beta d_\alpha^\beta A_i^\alpha = 0 \quad (6.3)$$

where we have used the notation [9]  $N_{IJ} = \frac{1}{4}(F_{IJ} + \bar{F}_{IJ})$  in terms of a homogenous function  $F(X^I)$  of degree two in  $X^I$ . Here the subscripts  $I, J, \dots$  stand for the

derivatives with respect to  $X^I, X^J, \dots$ , respectively. The function  $F(X^I)$  obeys the relations

$$\begin{aligned} F(X) &= \frac{1}{2}F_I(X)X^I, \quad F_I(X) = F_{IJ}(X)X^J, \\ F_{IJK}(X)X^K &= 0, \quad F_{IJK}(X) = -F_{IJKL}(X)X^L \end{aligned}$$

As a result, the  $S$ -supersymmetry parameters can be written down in terms of the  $N = 1/2$   $Q$ -supersymmetry parameter  $\bar{\varepsilon}_1$  as follows:

$$\begin{aligned} \eta_{1A} &= -(\bar{\varepsilon}_1 \sigma^\mu)_A \left[ \frac{1}{8}N_{IJ}(\Omega_1^I \sigma_\mu \bar{\Omega}^{1I} - \Omega_2^I \sigma_\mu \bar{\Omega}^{2J}) + d^\alpha{}_\beta (\zeta_\alpha \sigma_\mu \bar{\zeta}^\beta) \right], \\ \eta_{2A} &= -\frac{1}{4}(\bar{\varepsilon}_1 \sigma^\mu)_A N_{IJ} \Omega_2^I \sigma_\mu \bar{\Omega}^{1J}, \\ \bar{\eta}_{\dot{A}}^1 &= +2g d^\alpha{}_\beta \bar{\varepsilon}_{1\dot{A}} A_2^\alpha X_\beta{}^\gamma A^1{}_\gamma, \\ \bar{\eta}_{\dot{A}}^2 &= -2g d^\alpha{}_\beta \bar{\varepsilon}_{1\dot{A}} A_1^\alpha X_\beta{}^\gamma A^1{}_\gamma - d_\alpha{}^\beta \rho_{\beta\gamma} (\sigma^{\rho\lambda} \bar{\varepsilon}_1)_{\dot{A}} (\bar{\zeta}^\alpha \sigma_{\rho\lambda} \bar{\zeta}^\gamma) \end{aligned}$$

Similarly, as regards the chiral  $U(1)$  rotations, we find

$$\Lambda_A = \frac{i}{2} N_{IJ} \bar{\varepsilon}_1 \bar{\Lambda}^{IJ} X^J$$

The important part of the superconformal tensor calculus is a construction of the (superconformally) invariant actions. As regards the  $N = 2$  case, the invariant action for vector multiplets is given by eq. (3.9) of ref. [9], whereas the invariant action of hypermultiplets is given by eq. (3.29) of ref. [9]. The full invariant action for vector- and hyper-multiplets coupled to  $N = 2$  conformal supergravity is a sum of eq. (3.9), a Chern-Simons coupling (3.16), and eq. (3.29) of ref. [9]. We use the same  $N = 2$  superconformally invariant action as our starting point. Hence, we are still in a position to fix the algebraic field equations of the auxiliary fields that follow from the full action [9] after taking into account our constraint (Sec. 4).<sup>7</sup>

As regards the chiral  $U(1)$  gauge fields, we find

$$A_\mu = \frac{i}{2} N_{IJ} [X^I \hat{\partial}_\mu \bar{X}^J - (\hat{\partial}_\mu X^I) \bar{X}^J] - \frac{1}{8} N_{IJ} \bar{\Omega}^{iI} \sigma_\mu \Omega_i^J - d_\alpha{}^\beta \bar{\zeta}^\alpha \sigma_\mu \zeta_\beta \quad (6.4)$$

As regards the chiral  $SU(2)$  gauge fields, the  $\mathcal{V}_{\mu 2}^1$  is already fixed (Sec. 4) by the constraint (4.9) whose solution is

$$\mathcal{V}_{\mu 2}^1 = \frac{1}{X^0} (\psi_\mu^1 \Omega_2^0) \quad (6.5)$$

The remaining  $SU(2)$  gauge fields are given by (when  $i \neq 2$  and  $j \neq 1$ )

$$\mathcal{V}_{\mu i}^j = d^\beta{}_\alpha (\partial_\mu A_\beta^j A_i^\alpha - A_\beta^j \partial_\mu A_i^\alpha) - \frac{i}{2} N_{IJ} \bar{\Omega}^{jI} \sigma_\mu \Omega_i^J + \frac{i}{4} \delta_i^j N_{IJ} \bar{\Omega}^{kI} \sigma_\mu \Omega_k^J \quad (6.6)$$

---

<sup>7</sup>It is always assumed here that we are working in Euclidean space. Hence, the results of ref. [9] are to be reformulated in the Euclidean signature with the 2-component notation for spinors – see Sec. 3.

The  $T_{\mu\nu}^{+ij}$  after the  $C$ -deformation ( $F_{\mu\nu}^{0+} = C_{\mu\nu}$ ) is determined by the algebraic equation

$$\begin{aligned} N_{IJ}X^IX^JT_{\mu\nu}^{+ij}\varepsilon_{ij} - 4N_{I0}X^IC_{\mu\nu} - 4N_{I\tilde{J}}X^I\hat{F}_{\mu\nu}^{+\tilde{J}} \\ - 8d_\alpha^\beta\rho_{\beta\gamma}\bar{\zeta}^\alpha\sigma_{\mu\nu}\bar{\zeta}^\gamma - \frac{1}{4}\bar{F}_{IJK}X^K\varepsilon_{ij}\bar{\Omega}^{iI}\sigma_{\mu\nu}\bar{\Omega}^{jJ} = 0, \end{aligned} \quad (6.7)$$

whereas for the  $T_{\mu\nu}^-$  after the  $C$ -deformation ( $F_{\mu\nu}^{0-} = 0$ ) we find

$$\begin{aligned} N_{IJ}\bar{X}^I\bar{X}^JT_{\mu\nu}^-\varepsilon^{ij} - 4N_{I\tilde{J}}\bar{X}^I\hat{F}_{\mu\nu}^{--\tilde{J}} \\ + 8d_\alpha^\beta\rho^{\beta\gamma}\zeta_\gamma\sigma_{\mu\nu}\zeta_\alpha + \frac{1}{4}F_{IJK}\bar{X}^K\varepsilon^{ij}\Omega_i^I\sigma_{\mu\nu}\Omega_j^J = 0 \end{aligned} \quad (6.8)$$

Finally, the vector multiplet auxiliary fields obey the algebraic field equations

$$-\frac{1}{4}N_{IJ}Y_{ij}^J + gd_\alpha^\beta A^k_{\beta}\varepsilon_{ki}t_I^\alpha A_j^\gamma - \frac{1}{32}(F_{IJK}\Omega_i^J\Omega_j^K + \bar{F}_{IJK}\varepsilon_{ik}\varepsilon_{jl}\bar{\Omega}^{kJ}\bar{\Omega}^{lK}) = 0, \quad (6.9)$$

and

$$-\frac{1}{4}N_{IJ}Y^{ijJ} - gd_\alpha^\beta A^j_{\beta}\varepsilon^{ik}t_{I\alpha}^\gamma A_k^\beta + \frac{1}{32}(\bar{F}_{IJK}\bar{\Omega}_i^J\bar{\Omega}_j^K + \bar{F}_{IJK}\varepsilon^{ik}\varepsilon^{jl}\Omega_k^J\Omega_l^K) = 0 \quad (6.10)$$

## 7 Lagrangian

A derivation of the full Lagrangian of the new  $N = 1/2$  supergravity with vector- and hyper-multiplets is now fully straightforward, so we merely present our final result.

Let  $\hat{\partial}_\mu$  be the covariant derivative with respect to the non-abelian gauge transformations and local Lorentz rotations, but not w.r.t. the chiral  $U(1)$  and  $SU(2)$  rotations. The gravitino field  $\bar{\psi}_{\mu 2}$  is not independent, but a solution to eq. (4.7). In our Lagrangian  $\bar{\psi}_{\mu 2}$  is just the notation for  $\bar{\psi}_{\mu 2A} = -\frac{i}{2X^0}(\sigma_\mu\Omega_2^0)_A$ .

The  $N = 1/2$  supergravity Lagrangian has the following structure:

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{4\text{-fermi}} + \mathcal{L}_{\text{contact}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{F}} + \mathcal{L}_{\text{aux}}, \quad (7.1)$$

whose separate terms read as follows:

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{kin}} = & -\frac{1}{2}R - d_\alpha^\beta\hat{\partial}_\mu A^i_\beta\hat{\partial}^\mu A_i^\alpha + N_{IJ}\hat{\partial}_\mu X^I\hat{\partial}^\mu\bar{X}^J \\ & + ie^{-1}\varepsilon^{\mu\nu\rho\sigma}\psi_\mu^i\sigma_\nu\hat{\partial}_\rho\bar{\psi}_{\sigma i} \\ & - \frac{i}{4}N_{IJ}\bar{\Omega}^{iI}\sigma^\mu\hat{\partial}_\mu\Omega_i^J + \frac{i}{4}N_{IJ}\Omega_i^I\sigma^\mu\hat{\partial}_\mu\bar{\Omega}^{iJ} \\ & + 2id_\alpha^\beta\bar{\zeta}^\alpha\sigma^\mu\hat{\partial}_\mu\zeta_\beta + 2id_\alpha^\beta(\hat{\partial}_\mu\bar{\zeta}^\beta)\sigma^\mu\zeta_\alpha, \end{aligned}$$

$$\begin{aligned}
e^{-1}\mathcal{L}_{4\text{-fermi}} = & + \frac{1}{16}N_{IJ}e^{-1}\varepsilon^{\mu\nu\rho\sigma}(\bar{\psi}_{\mu i}\sigma_{\nu}\Omega_j\varepsilon^{ij})(\bar{\psi}_{\rho k}\sigma_{\sigma}\Omega_l^J\varepsilon^{kl}) \\
& + \frac{1}{16}N_{IJ}e^{-1}\varepsilon^{\mu\nu\rho\sigma}(\bar{\Omega}^{jI}\sigma_{\nu}\psi_{\mu}^i\varepsilon_{ij})(\bar{\Omega}^{lK}\sigma_{\sigma}\psi_{\rho}^k\varepsilon_{kl}) \\
& + \frac{1}{8}N_{IJ}e^{-1}\varepsilon^{\mu\nu\rho\sigma}(\bar{\psi}_{\mu i}\bar{\psi}_{\nu j}\varepsilon^{ij})\left[-i\bar{\psi}_{\rho k}\sigma_{\sigma}\Omega_l^IX^J - \frac{1}{2}\bar{\psi}_{\rho k}\bar{\psi}_{\sigma l}X^IX^J\right]\varepsilon^{kl} \\
& - \frac{1}{8}N_{IJ}e^{-1}\varepsilon^{\mu\nu\rho\sigma}(\psi_{\mu}^i\psi_{\nu}^j\varepsilon_{ij})\left[-i\bar{\Omega}^{lI}\sigma_{\sigma}\psi_{\rho}^k\bar{X}^J + \frac{1}{2}\psi_{\rho}^k\psi_{\sigma}^l\varepsilon_{kl}\bar{X}^I\bar{X}^J\right]\varepsilon_{kl} \\
& + \frac{i}{48}F_{IJK}(\bar{\psi}_{i\mu}\sigma^{\mu}\Omega_k^I)(\Omega_l^J\Omega_j^K)\varepsilon^{ij}\varepsilon^{kl} \\
& - \frac{i}{48}\bar{F}_{IJK}(\bar{\Omega}^{kI}\sigma^{\mu}\psi_{\mu}^i)(\bar{\Omega}^{jK}\bar{\Omega}^{lJ})\varepsilon_{ij}\varepsilon_{kl} \\
& - d_{\alpha}^{\beta}(\bar{\zeta}^{\alpha}\sigma^{\mu}\sigma^{\nu}\bar{\psi}_{\mu i})(\zeta_{\beta}\psi_{\nu}^i + \rho_{\beta\gamma}\bar{\zeta}^{\gamma}\bar{\psi}_{\nu j}\varepsilon^{ij}) \\
& + d_{\beta}^{\alpha}(\psi_{\mu}^i\sigma^{\nu}\sigma^{\mu}\zeta_{\alpha})(\bar{\psi}_{\nu i}\bar{\zeta}^{\beta} + \rho^{\beta\gamma}\psi_{\nu}^j\zeta_{\gamma}\varepsilon_{ij}) \\
& - \frac{1}{192}F_{IJKL}(\Omega_i^I\Omega_k^J)(\Omega_l^K\Omega_l^L)\varepsilon^{ij}\varepsilon^{kl} \\
& - \frac{1}{192}\bar{F}_{IJKL}(\bar{\Omega}^{iI}\bar{\Omega}^{kJ})(\bar{\Omega}^{jK}\bar{\Omega}^{lL})\varepsilon_{ij}\varepsilon_{kl} ,
\end{aligned}$$

$$\begin{aligned}
e^{-1}\mathcal{L}_{\text{contact}} = & + \frac{i}{4}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu i}\sigma_{\nu}\psi_{\rho}^iN_{IJ}[X^I(\hat{\partial}_{\sigma}\bar{X}^J) - (\hat{\partial}_{\sigma}X^I)\bar{X}^J] \\
& + \frac{i}{2}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu i}\sigma_{\nu}\psi_{\sigma}^jd_{\alpha}^{\beta}(A_j^{\alpha}\hat{\partial}_{\sigma}A_{\beta}^i - A_{\beta}^i\hat{\partial}_{\sigma}A_j^{\alpha}) \\
& - \frac{1}{2}N_{IJ}\psi_{\mu}^i\sigma^{\nu}\sigma^{\mu}\Omega_i^J\hat{\partial}_{\nu}\bar{X}^I - \frac{1}{2}N_{IJ}\bar{\Omega}^{iJ}\sigma^{\mu}\sigma^{\nu}\bar{\psi}_{\mu i}\hat{\partial}_{\nu}X^I \\
& + 2d_{\alpha}^{\beta}\psi_{\mu}^i\sigma^{\nu}\sigma^{\mu}\zeta_{\beta}\hat{\partial}_{\nu}A_i^{\alpha} + 2d_{\alpha}^{\beta}\bar{\zeta}^{\beta}\sigma^{\mu}\sigma^{\nu}\bar{\psi}_{\mu i}\hat{\partial}_{\nu}A_i^{\alpha} \\
& + \frac{1}{8}N_{IJ}(\bar{\Omega}^{iI}\sigma^{\mu}\sigma^{\nu}\bar{\psi}_{\mu i})(\psi_{\nu}^j\Omega_j^J) - \frac{1}{8}N_{IJ}(\psi_{\mu}^i\sigma^{\nu}\sigma^{\mu}\Omega_i^I)(\bar{\Omega}^{jJ}\bar{\psi}_{\nu j}) \\
& + \frac{i}{16}F_{IJK}\Omega_i^I\sigma^{\mu}\bar{\Omega}^{iK}\hat{\partial}_{\mu}X^J + \frac{i}{16}\bar{F}_{IJK}\Omega_i^K\sigma^{\mu}\bar{\Omega}^{iI}\hat{\partial}_{\mu}\bar{X}^J ,
\end{aligned}$$

$$\begin{aligned}
e^{-1}\mathcal{L}_{\text{gauge}} = & - \frac{i}{6}gC_{I,JK}e^{-1}\varepsilon^{\mu\nu\rho\sigma}W_{\mu}^IW_{\nu}^J\left(\partial_{\rho}W_{\sigma}^K - \frac{3}{2}gf_{LM}^KW_{\rho}^LW_{\sigma}^M\right) \\
& + 4g^2d_{\alpha}^{\beta}A_{\beta}^i\bar{X}^{\alpha}\gamma X_{\delta}^{\gamma}A_i^{\delta} - g^2N_{IJ}f_{KL}^I\bar{X}^KX^Lf_{MN}^J\bar{X}^MX^N \\
& - \frac{1}{2}gN_{IJ}\Omega_i^If_{KL}^J\bar{X}^K\Omega_j^L\varepsilon^{ij} + \frac{1}{2}gN_{IJ}\bar{\Omega}^{jL}X^Kf_{KL}^J\bar{\Omega}^{iI}\varepsilon_{ij} \\
& + 4gd_{\alpha}^{\beta}A_{\beta}^i\bar{\Omega}^{j\alpha}\gamma\bar{\zeta}^{\gamma}\varepsilon_{ij} - 4gd_{\alpha}^{\beta}A_i^{\beta}\zeta_{\gamma}\Omega_{j\alpha}^{\gamma}\varepsilon^{ij} \\
& - 4gd_{\alpha}^{\beta}\rho_{\beta\gamma}\bar{\zeta}^{\alpha}\bar{X}^{\gamma}\bar{\zeta}^{\delta} + 4gd_{\alpha}^{\beta}\rho^{\beta\gamma}\zeta_{\delta}X_{\gamma}^{\delta}\zeta_{\alpha} \\
& - \frac{i}{2}gN_{IJ}\psi_{\mu}^i\sigma^{\mu}\bar{\Omega}^{jI}\varepsilon_{ij}f_{KL}^JX^K\bar{X}^L - \frac{i}{2}gN_{IJ}\Omega_j^I\sigma^{\mu}\bar{\psi}_{\mu i}\varepsilon^{ij}f_{KL}^J\bar{X}^KX^L \\
& + 4igd_{\alpha}^{\beta}\psi_{\mu}^i\sigma^{\mu}\bar{\zeta}^{\gamma}A_{\beta}^j\bar{X}^{\alpha}\gamma\varepsilon_{ij} + 4igd_{\alpha}^{\beta}\zeta_{\gamma}\sigma^{\mu}\bar{\psi}_{\mu i}X_{\alpha}^{\gamma}A_j^{\beta}\varepsilon^{ij} \\
& + igd_{\alpha}^{\beta}\psi_{\mu}^i\sigma^{\mu}\bar{\Omega}^{k\alpha}\gamma A_{\beta}^jA_k^{\gamma}\varepsilon_{ij} + igd_{\alpha}^{\beta}\Omega_{k\alpha}^{\gamma}\sigma^{\mu}\bar{\psi}_{\mu i}A_k^{\gamma}A_j^{\beta}\varepsilon^{ij} \\
& + 2gd_{\alpha}^{\beta}\psi_{\mu}^i\sigma^{\mu\nu}\psi_{\nu}^jA_i^{\alpha}A_k^{\gamma}\bar{X}_{\beta}^{\gamma}\varepsilon_{jk} - 2gd_{\alpha}^{\beta}\bar{\psi}_{\nu j}\sigma^{\mu\nu}\bar{\psi}_{\mu i}X_{\beta}^{\gamma}A_k^{\gamma}A_i^{\alpha}\varepsilon^{jk} ,
\end{aligned}$$

where  $C_{I,JK}$  are the real coefficient functions defined in terms of the input function  $F(X)$  by considering the non-abelian gauge transformations of the latter (with the gauge coupling constant  $g$ ) [9]:

$$\delta F = gF_Jf_{IK}^JX^K\Lambda^I \equiv ig\Lambda^IC_{I,JK}X^JX^K \quad (7.2)$$

The coefficient functions  $C_{I,JK}$  are symmetric in the last two indices, and obey a relation [9]

$$C_{I,JK} + C_{J,KL} + C_{K,LI} = 0 \quad (7.3)$$

The  $\mathcal{L}_F$  part of the Lagrangian (7.1) is given by

$$\begin{aligned} e^{-1}\mathcal{L}_F = & -\frac{1}{4}\left[N_{I0}C^{\mu\nu} + N_{I\tilde{J}}\tilde{F}^{\mu\nu\tilde{J}}N_{IJ}\tilde{G}^{\mu\nu J}\right]G_{\mu\nu}^I \\ & + \frac{1}{8}\left[N_{00}(C^{\mu\nu} + G^{\mu\nu 0}) + N_{0\tilde{J}}\hat{F}^{\mu\nu\tilde{J}}\right](C_{\mu\nu} + G_{\mu\nu}^0) \\ & + \frac{1}{8}\left[N_{\tilde{I}0}(C^{\mu\nu} + G^{\mu\nu 0}) + N_{\tilde{I}\tilde{J}}\hat{F}^{\mu\nu\tilde{J}}\right]\hat{F}_{\mu\nu}^{\tilde{I}} \\ & + \frac{1}{32}\left[\overline{F}_{IJ0}C_{\mu\nu} + \overline{F}_{IJ\tilde{K}}F_{\mu\nu}^{+\tilde{K}} + \overline{F}_{IJK}G_{\mu\nu}^{+K}\right](\overline{\Omega}^{iI}\sigma^{\mu\nu}\overline{\Omega}^{jJ}\varepsilon_{ij}) \\ & - \frac{1}{32}\left[F_{IJ\tilde{K}}F^{-\tilde{K}} + F_{IJK}G_{\mu\nu}^{-K}\right](\Omega_j^J\sigma^{\mu\nu}\Omega_i^I\varepsilon^{ij}) \\ & - \frac{1}{32}(F_{00} - \overline{F}_{00})C_{\mu\nu}C^{\mu\nu} - \frac{1}{16}(F_{\tilde{I}0} - \overline{F}_{\tilde{I}0})F_{\mu\nu}^{\tilde{I}}C^{\mu\nu} \\ & - \frac{1}{64}(F_{\tilde{I}\tilde{J}} - \overline{F}_{\tilde{I}\tilde{J}})e^{-1}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^{\tilde{I}}F_{\rho\sigma}^{\tilde{J}} \end{aligned}$$

where we have introduced the non-abelian vector field strength

$$F_{\mu\nu}^{\tilde{I}} = \partial_\mu W_\nu^{\tilde{I}} - \partial_\nu W_\mu^{\tilde{I}} - gf_{\tilde{J}\tilde{K}}^{\tilde{I}}W_\mu^{\tilde{J}}W_\nu^{\tilde{K}} \quad (7.4)$$

and the book-keeping notation

$$G_{\mu\nu}^I := -\frac{i}{2}\left[\left(\overline{\Omega}^{iI}\sigma_\mu\psi_\nu^j\varepsilon_{ij} - \Omega_i^I\sigma_\mu\overline{\psi}_{\nu j}\varepsilon^{ij} - (\mu \leftrightarrow \nu)\right)\right] + \left(\overline{X}^I\psi_\mu^i\psi_\nu^j\varepsilon_{ij} - X^I\overline{\psi}_{\mu i}\overline{\psi}_{\nu j}\varepsilon^{ij}\right) \quad (7.5)$$

Finally, the last term  $\mathcal{L}_{\text{aux}}$  in eq. (7.1) is given by

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{aux}} = & -A_\mu A^\mu - \frac{1}{4}\mathcal{V}_\mu^i{}_j\mathcal{V}^\mu{}_j{}^i + \frac{1}{8}N_{IJ}Y_{ij}^IY^{ijJ} \\ & - \frac{1}{64}N_{IJ}X^IX^J(T_{\mu\nu ij}^+\varepsilon^{ij})^2 - \frac{1}{64}N_{IJ}\overline{X}^I\overline{X}^J(T_{\mu\nu}^{-ij}\varepsilon_{ij})^2 \end{aligned}$$

The gravitino field  $\psi_\mu^2$  enters the Lagrangian (7.1) algebraically, so it may be eliminated via its non-propagating field equation. The bosonic part of the matrix multiplying  $\psi_\mu^2$  in its field equation has an inverse, due to the identity

$$(2\sigma_{\rho\lambda}{}^C{}_A - g_{\rho\lambda}\delta^C{}_A - \frac{1}{3}\sigma_\rho^{C\dot{D}}\sigma_{\lambda\dot{D}A})\sigma^{\lambda\mu A}{}_B = +\delta_\rho^\mu\delta^C{}_B \quad (7.6)$$

so that there is a unique solution for  $\psi_\mu^2$ .

## 8 Conclusion

We formulated the  $N = 1/2$  supergravity coupled to both vector and scalar matter multiplets in four Euclidean dimensions. The gauge field of the local  $N = (0, \frac{1}{2})$  supersymmetry is given by a single chiral gravitino. The  $C$ -deformation, originally introduced in the context of supersymmetric D-branes with RR-type fluxes, was the main tool of our purely field-theoretical construction. The new matter-coupled  $N = 1/2$  supergravity with matter is not invariant under the Euclidean analogue of Lorentz rotations in four dimensions, due to the explicit presence of a fixed (self-dual) graviphoton background. In addition, the Lagrangian we constructed (Sec. 7) is not Hermitean, thus hampering immediate physical applications of the proposed new supergravity theory. However, those are the common problems of all recently constructed  $N = 1/2$  supersymmetric field theories, either with rigid or local  $N = 1/2$  supersymmetry.

When compared to our earlier construction of the pure  $N = 1/2$  supergravity without matter [11], in the matter-coupled  $N = 1/2$  supergravity we observe no formation of the gravitino condensate. Instead, we got one more constraint on the gravitini in terms of the matter fields.

Being the first construction of that type, our  $N = 1/2$  supergravity with matter is unlikely to be the most general one having a local  $N = 1/2$  supersymmetry. The use of the  $N = 2$  superconformal tensor calculus was essential in our construction, while we still have the vector multiplet scalars parameterizing a special Kähler manifold, and the hypermultiplet scalars parameterizing a quaternionic (projective) manifold or its quaternionic quotient. It is rather straightforward to generalize our results to the case of arbitrary (quaternionic) hypermultiplet couplings by using the same  $N=2$  superconformal calculus and the results of ref. [16]. However, we cannot exclude the existence of a much larger class of the invariant actions with merely  $N = 1/2$  local supersymmetry, which are not derivable from the  $N = 2$  invariant actions we used. It would be interesting to find such additional invariants for a construction of the most general  $N = 1/2$  supergravity matter couplings.

We are unaware of any supergravity model to be constructed in a (curved) non-anticommutative superspace, so a relation of our  $N = 1/2$  matter-coupled supergravity to the non-anticommutative superspace remains unclear to us.

It may also be of interest to study the conditions of further (spontaneous) breaking of local  $N = 1$  supersymmetry by analyzing the vacuum expectation values of the supersymmetry transformations of the fermionic fields.<sup>8</sup>

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<sup>8</sup>See e.g., ref. [17], as regards partial breaking of  $N = 2$  local supersymmetry.



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## A notation for hypermultiplets

We follow the notation introduced in ref. [9]. In particular, the hypermultiplets  $(A_i^\alpha, \zeta^\alpha)$  belong to a representation of the Yang-Mills group. The scalars  $A_i^\alpha$  obey a reality condition

$$A^i_\alpha \equiv \overline{A_i^\alpha} = \epsilon^{ij} \rho_{\alpha\beta} A_j^\beta \quad (\text{A.1})$$

where the matrices  $\rho_{\alpha\beta}$  are used for raising and lowering of greek indices. A consistency requires

$$\rho_{\alpha\beta} \rho^{\beta\gamma} = -\delta_\alpha^\gamma \quad (\rho^{\alpha\beta} \equiv \overline{\rho_{\alpha\beta}}) \quad (\text{A.2})$$

When writing the action, it is convenient to introduce a matrix  $d^\alpha_\beta$  as

$$d^\alpha_\beta := -\eta^{[\alpha\gamma]} \rho_{\gamma\beta} \quad (\text{A.3})$$

where  $\eta^{\alpha\gamma}$  is the real multiplication tensor in the sense

$$\eta_{\alpha\beta} = \rho_{\gamma\alpha} \eta^{\gamma\delta} \rho_{\delta\beta}, \quad (\eta^{\alpha\beta} \equiv \overline{\eta_{\alpha\beta}}) \quad (\text{A.4})$$

The matrix  $d^\alpha_\beta$  has the properties

$$\overline{d^\alpha_\beta} \equiv d_\alpha^\beta = d^\beta_\alpha \quad (\text{Hermitean}) \quad d_\alpha^\beta = \rho_{\gamma\alpha} \rho^{\delta\beta} d^\gamma_\delta \quad (\text{quaternionic}) \quad (\text{A.5})$$

## B N=2 superconformal transformation laws

In this Appendix we summarize the relevant part of the  $N = 2$  superconformal transformation laws [9] in our 2-component spinor notation, in four Euclidean dimensions:

(i) as regards the  $N = 2$  *Weyl* multiplet (physical) components,

$$\delta e_\mu^a = -i\varepsilon^i \sigma^a \bar{\psi}_{\mu i} - i\bar{\varepsilon}_i \sigma^a \psi_\mu^i, \quad (\text{B.1})$$

$$\begin{aligned} \delta \psi_\mu^{iA} = & + 2 \left[ \partial_\mu \varepsilon^{iA} - \frac{1}{2} \omega_\mu^{mn} \sigma_{mn}^A \varepsilon^{iB} + \frac{1}{2} i A_\mu \varepsilon^{iA} \right] + \mathcal{V}_\mu^i \varepsilon^{jA} \\ & - \frac{1}{2} i T_{\mu m}^{-ij} (\sigma^m \bar{\varepsilon}_j)^A + i (\sigma_\mu \bar{\eta}^i)^A - \frac{1}{2} i \Lambda_A \psi_\mu^{iA} + \Lambda^i_j \psi_\mu^{jA}, \end{aligned} \quad (\text{B.2})$$

$$\begin{aligned} \delta \bar{\psi}_{\mu i \dot{A}} = & + 2 \left[ \partial_\mu \bar{\varepsilon}_{i \dot{A}} - \frac{1}{2} \omega_\mu^{mn} \sigma_{mn}^{\dot{B} \dot{A}} \bar{\varepsilon}_{i \dot{B}} - \frac{1}{2} i \bar{A}_\mu \bar{\varepsilon}_{i \dot{A}} \right] + \mathcal{V}_{\mu i}^{\dot{j}} \bar{\varepsilon}_{j \dot{A}} \\ & + \frac{1}{2} i T_{\mu m i j}^+ (\sigma^m \varepsilon^j)_{\dot{A}} - i (\sigma_\mu \eta_i)_{\dot{A}} + \frac{1}{2} i \Lambda_A \bar{\psi}_{\mu i \dot{A}} + \Lambda_i^j \bar{\psi}_{\mu j \dot{A}} \end{aligned} \quad (\text{B.3})$$

(ii) as regards the  $N = 2$  *vector* multiplet components,

$$\delta X^I = -\varepsilon^i \Omega_i^I - i \Lambda_A X^I \quad (\text{B.4})$$

$$\delta \bar{X}^I = +\bar{\varepsilon}_i \bar{\Omega}^{iI} + i \Lambda_A \bar{X}^I \quad (\text{B.5})$$

$$\delta W_\mu^I = -i \bar{\varepsilon}_i \sigma_\mu \Omega_j^I \varepsilon^{ij} + i \varepsilon^i \sigma_\mu \bar{\Omega}^{jI} \varepsilon_{ij} + 2 \varepsilon^i \psi_\mu^j \bar{X}^I \varepsilon_{ij} - 2 \bar{\varepsilon}_i \bar{\psi}_{\mu j} X^I \varepsilon^{ij}, \quad (\text{B.6})$$

$$\begin{aligned} \delta \Omega_i^{IA} = & -2i (\sigma^\lambda \bar{\varepsilon}_i)^A [(\partial_\lambda + i A_\lambda) X^I - g f_{JK}^I W_\lambda^J X^K - \frac{1}{2} \psi_\lambda^k \Omega_k^I] \\ & + Y_{ij}^I \varepsilon^{jA} + \sigma^{mnA}{}_B \mathcal{F}_{mn}^{-I} \varepsilon_{ij} \varepsilon^{jB} - 2g f_{JK}^I X^J \bar{X}^K \varepsilon_{ij} \varepsilon^{jA} \\ & + 2X^I \eta_i^A - \frac{1}{2} i \Lambda_A \Omega_i^{IA} + \Lambda_i^j \Omega_j^{IA}, \end{aligned} \quad (\text{B.7})$$

$$\begin{aligned} \delta \bar{\Omega}^{iI \dot{A}} = & -2i (\varepsilon^i \sigma^\lambda)_{\dot{A}} [(\partial_\lambda - i \bar{A}_\lambda) \bar{X}^I - g f_{JK}^I W_\lambda^J \bar{X}^K + \frac{1}{2} \bar{\psi}_{\lambda k} \bar{\Omega}^{kI}] \\ & + Y^{ikI} \bar{\varepsilon}_k^{\dot{A}} - \bar{\varepsilon}_k^{\dot{B}} \sigma^{mn}{}_{\dot{B}}{}^{\dot{A}} \mathcal{F}_{mn}^{+I} \varepsilon^{ik} - 2g f_{JK}^I \bar{X}^J X^K \varepsilon^{ik} \bar{\varepsilon}_k^{\dot{A}} \\ & + 2\bar{X}^I \bar{\eta}^{i \dot{A}} + \frac{1}{2} i \Lambda_A \bar{\Omega}^{iI \dot{A}} + \Lambda_i^k \bar{\Omega}^{kI \dot{A}} \end{aligned} \quad (\text{B.8})$$

(iii) and as regards *hypermultiplets*,

$$\delta A_i^\alpha = -2\bar{\zeta}^\alpha \bar{\varepsilon}_i - 2\rho^{\alpha\beta} \varepsilon_{ij} \zeta_\beta \varepsilon^j + \Lambda_i^j A_j^\alpha, \quad (\text{B.9})$$

$$\delta A^i_\alpha = -2\varepsilon^i \zeta_\alpha - 2\rho_{\alpha\beta} \varepsilon^{ij} \bar{\varepsilon}_j \bar{\zeta}^\beta + \Lambda^i_j A^j_\alpha, \quad (\text{B.10})$$

$$\delta \zeta_{\alpha A} = +i (\bar{\varepsilon}_i \sigma^\mu)_A \bar{D}_\mu A_\alpha^i + 2g A^i_\beta \bar{X}^\beta \varepsilon_{ij} \varepsilon_A^j + A^i_\alpha \eta_{iA} + \frac{i}{2} \Lambda_A \zeta_{\alpha A} \quad (\text{B.11})$$

$$\delta \bar{\zeta}_A^\alpha = +i (\sigma^\mu \varepsilon^i)_{\dot{A}} D_\mu A_i^\alpha + 2g X^\alpha_\beta A_i^\beta \varepsilon^{ij} \bar{\varepsilon}_{j \dot{A}} + A_i^\alpha \bar{\eta}_{\dot{A}}^i - \frac{i}{2} \Lambda_A \bar{\zeta}_A^\alpha, \quad (\text{B.12})$$

where we have used the following abbreviations:

$$\mathcal{F}_{\mu\nu}^I := \hat{F}_{\mu\nu}^I - \frac{1}{4} X^I T_{\mu\nu ij} \varepsilon^{ij}, \quad \hat{F}_{\mu\nu}^I := F_{\mu\nu}^I + G_{\mu\nu}^I, \quad (\text{B.13})$$

$$D_\mu A_i^\alpha := \partial_\mu A_i^\alpha + \frac{1}{2} \mathcal{V}_{\mu i}^j A_j^\alpha - g W_\mu^\alpha{}_\beta A_i^\beta + \bar{\zeta}^\alpha \bar{\psi}_{\mu i} + \rho^{\alpha\beta} \varepsilon_{ij} \zeta_\beta \psi_\mu^j, \quad (\text{B.14})$$

$$\bar{D}_\mu A^i_\alpha = \partial_\mu A^i_\alpha + \frac{1}{2} A^j_\alpha \mathcal{V}_\mu^i{}_j - g A^i_\beta W_{\mu\alpha}^\beta + \psi_\mu^i \zeta_\alpha + \rho_{\alpha\beta} \varepsilon^{ij} \bar{\psi}_{\mu j} \bar{\zeta}^\beta \quad (\text{B.15})$$

As for the auxiliary fields, we have e.g.,

$$\begin{aligned} \delta \mathcal{V}_\mu^i{}_j = & -3i(\varepsilon^i \sigma_\mu \bar{\chi}_j - \varepsilon^{il} \varepsilon_{jm} \chi^m \sigma_\mu \bar{\varepsilon}_l) + 2\left(\varepsilon^i \phi_{\mu j} - \psi_\mu^i \eta_j + \varepsilon^{il} \varepsilon_{jm} (\bar{\varepsilon}_l \bar{\phi}_\mu^m - \bar{\psi}_\mu \bar{\eta}^m)\right) \\ & - \frac{1}{2} \delta^i{}_j \left[ -3i(\varepsilon^k \sigma_\mu \bar{\chi}_k - \chi^k \sigma_\mu \bar{\varepsilon}_k) + 2(\varepsilon^k \phi_{\mu k} + \bar{\varepsilon}_k \bar{\phi}_\mu^k - \psi_\mu^k \eta_k - \bar{\psi}_{\mu k} \bar{\eta}^k) \right] \end{aligned} \quad (\text{B.16})$$

## C decomposition rules

After the  $C$ -deformation (Sec. 3) and the gauge-fixing of the truly superconformal symmetries (Sec. 6), the  $Q$ -supersymmetry transformations are modified by the compensating  $S$ -supersymmetry-, chiral  $U(1)$ - and chiral  $SU(2)$ - transformations, whose parameters are given by

$$\begin{aligned} \eta_{iA} = & + 2g d^\alpha{}_\beta \varepsilon_{ij} \varepsilon_A^k A_k^\gamma \bar{X}^\beta{}_\gamma A^j{}_\alpha + d^\alpha{}_\beta \rho^{\beta\gamma} \varepsilon_{ij} (\varepsilon^j \sigma^{\rho\lambda})_A (\zeta^\gamma \sigma_{\rho\lambda} \zeta_\alpha) \\ & - (\bar{\varepsilon}_j \sigma^\mu)_A \left[ \frac{1}{4} N_{IJ} (\Omega_i^I \sigma_\mu \bar{\Omega}^{jJ} - \frac{1}{2} \delta_i^j \Omega_k^I \sigma_\mu \bar{\Omega}^{kJ}) + \delta_i^j d^\alpha{}_\beta (\zeta_\alpha \sigma_\mu \bar{\zeta}^\beta) \right], \end{aligned} \quad (\text{C.1})$$

$$\begin{aligned} \bar{\eta}_{\dot{A}}^i = & + 2g d^\alpha{}_\beta \varepsilon^{ij} \bar{\varepsilon}_{k\dot{A}} A_j^\alpha X_\beta{}^\gamma A_k^\gamma + d_\alpha{}^\beta \rho_{\beta\gamma} \varepsilon^{ij} (\sigma^{\rho\lambda} \bar{\varepsilon}_j)_{\dot{A}} (\bar{\zeta}^\alpha \sigma_{\rho\lambda} \bar{\zeta}^\gamma) \\ & + (\sigma^\mu \varepsilon^j)_{\dot{A}} \left[ \frac{1}{4} N_{IJ} (\Omega_j^I \sigma_\mu \bar{\Omega}^{iJ} - \frac{1}{2} \delta_j^i \Omega_k^I \sigma_\mu \bar{\Omega}^{kJ}) + \delta_j^i d_\alpha{}^\beta (\bar{\zeta}^\alpha \sigma_\mu \zeta_\beta) \right], \end{aligned} \quad (\text{C.2})$$

$$\Lambda_A = + \frac{1}{2} i N_{IJ} (-\varepsilon^i \Lambda_i^I \bar{X}^J + \bar{\varepsilon}_i \bar{\Lambda}^{iI} X^J), \quad (\text{C.3})$$

$$\begin{aligned} \Lambda_i^j = & + 2\sqrt{\frac{-2}{c}} \left( d_\alpha{}^\beta [B^j{}_\beta (\bar{\xi}^\alpha \bar{\varepsilon}_i) - \frac{1}{2} \delta_i^j B^k{}_\beta (\bar{\xi}^\alpha \bar{\varepsilon}_k)] \right. \\ & \left. - d^\alpha{}_\beta [B_i{}^\beta (\xi_\alpha \varepsilon^j) - \frac{1}{2} \delta_i^j B_k{}^\beta (\xi_\alpha \varepsilon^k)] \right), \end{aligned} \quad (\text{C.4})$$

where the convenient parametrization is given by [9]

$$\begin{aligned} \Lambda_i^I &:= \Omega_i^I - Z^I \Omega_i^0 = \begin{pmatrix} \Lambda_{iA}^I \\ 0 \end{pmatrix}, \quad Z^I := \frac{X^I}{X^0}, \quad \xi^\alpha := \zeta^\alpha - B_a{}^\alpha \zeta^a = \begin{pmatrix} 0 \\ \bar{\xi}_A^\alpha \end{pmatrix}, \\ c &:= -4(A_i^a A_a^i)^{-1} = d_\alpha{}^\beta B_a{}^\alpha B_\beta^a, \quad B_a{}^\alpha := A^{-1i}{}_a A_i{}^\alpha, \\ a, b &= 1, 2, \quad \alpha, \beta = 3, \dots, 2r. \end{aligned} \quad (\text{C.5})$$

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